1995108069

N95-14503

20092 P. 10

Asteroseismology – Ground Based Efforts and the Need for Space Observations

Ronald L. Gilliland¹

Space Telescope Science Institute², 3700 San Martin Drive, Baltimore, MD 21218

Abstract. Detection of the oscillations expected to be present on solar-like stars is very difficult. Photometric observations from the ground suffer from two problems: 1) an atmospheric scintillation noise that drops only slowly with telescope aperture size, and 2) mode frequency spacings that require nearly continuous observations over at least several days for resolution. I will review the very limited possibilities for asteroseismology of solar-like stars from ground-based photometric observations. FRESIP could provide an excellent opportunity for pursuing asteroseismology observations of a far richer nature than can be contemplated from the ground.

1. Introduction

The companion paper in this volume by Tim Brown (1993) will have introduced the science of asteroseismology. Through the detection and detailed quantification of stellar oscillations on a large number of different stars we should be able to open a fundamentally new and important chapter in stellar astrophysics. The information that may be gained from asteroseismology should allow a useful confrontation with stellar structure and evolution theory.

In this paper I will concentrate on reviewing the results from a recent large scale photometry campaign (Gilliland, et al. 1993) directed toward detection of oscillations on subgiant stars in M67. I will emphasize that although the study of oscillations on solar-like stars using ground based photometry is not hopeless, it is very difficult and will remain of limited utility. Since the observational requirements of the FRESIP project so nearly match those for asteroseismology, the latter is an ideal candidate for auxiliary science with FRESIP. In §2 I will outline the basic observational requirements for detection of stellar oscillations. Section 3 will be devoted to a discussion of the largest ground-based photometry campaign to date. The capabilities and limitations of a possible future ground-based observing campaign making use of 10-12 of the worlds largest telescopes for a full week will be the topic of §4. (Parts of §3 and 4 have been adopted from

¹Guest Observer, Kitt Peak National Observatory, National Optical Astronomical Observatories, Operated by AURA, Inc., under contract with the National Science Foundation.

²Operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Aeronautics and Space Administration.

a previous conference contribution: Gilliland 1993.) The capabilities of FRESIP to support asteroseismology will be discussed in §5. A summary will be provided that includes a note that planetary detection (via photometry) is probably even more difficult to pursue from the ground than is asteroseismology.

2. Observational Requirements for Detection

Photometric observations acquired through the Earth's atmosphere have a (probably irreducible) noise component from atmospheric scintillation (e.g., see Young 1967; 1974). The atmospheric scintillation has been parameterized by Young as:

$$\delta I/I = 0.09 D^{-2/3} X^{1.75} \exp(-h/h_o) / (2t_{int})^{1/2}$$
 (1)

where I is the intensity, D is the telescope aperture diameter in cm, X is the airmass, h is the telescope altitude relative to the atmospheric scale height h_o (~ 8000 m) and the integration time in seconds is t_{int} . Unfortunately this equation exhibits only a weak fall off with telescope aperture size and a fairly strong increase with airmass. Even with the largest telescopes at high altitude, atmospheric scintillation strongly limits the photometric precisions that could otherwise be obtained.

On a 4-m telescope Eq. (1) predicts a noise floor of $\sim 215~\mu mag$ per minute of observation when averaged over an 8 hour window. Stars brighter than $m_B \sim 12$ will have Poisson errors less than this when observed with efficient detectors and a wide-band photometric filter. Thus in simple terms of limiting noise all stars with $m_B \leq 12$ are equal for 4-m class telescopes. (This assumes that other sources of noise such as atmospheric transparency variations can be controlled. In practice this further restricts candidate stars to fields with good ensembles within a small field of view.)

The number of observations required to detect a coherent oscillation of amplitude, A, given a time series of precision (rms), ϵ , may be written (Scargle 1982):

$$N_{obs} = 16(\epsilon/A)^2 \tag{2}$$

where we have assumed a significance level of $4\text{-}\sigma$ is desired. Evaluation of Eq. (2) for the Sun observed as a star with 4-m telescopes quickly points out the fundamental problem inherent in ground-based photometry to detect stellar oscillations: the signal is small compared to the noise. Specifically we might assume that a 4-m telescope (or network of such) could deliver 300 μ mag precision per minute if other noise sources end up equaling scintillation. With the solar amplitude of about 3 μ mag Eq. (2) requires 160,000 observations, or 111 full observing days, for detection. To obtain 111 full observing days given average weather conditions would require simultaneous allocation of time for the full period on six telescopes well separated from each other. Even with 10-m telescopes at 4,000m altitude it would take 35 (full) days to (just marginally in a quantitatively useful sense) detect oscillations on a strict solar analogue. Clearly ground-based photometry does not seem useful in these cases.

For ground-based photometry to have a decent chance of success stars with more favorable amplitudes must be selected. Christensen-Dalsgaard and Frandsen (1983) have predicted based on theory that F-G subgiants should have amplitudes of up to about five times solar. At this more favorable amplitude 4-m class telescopes might allow detection in 5 days (again this assumes full 24 hour days), while ideal 10-m class telescopes might do the same in less than two days.

Although five full days, which in practice requires a collaboration involving six sites all assigned for the full time, on 4-m class telescopes is a major effort, it is not beyond the realm of feasibility. Gilliland and Brown (1992) provided detailed predictions of what such a 4-m project might accomplish and selected a nearly ideal (from both observational and theoretical constraints) ensemble of 11 stars in the old open cluster M67.

Before moving to a discussion of the 4-m network observing campaign I need to point out two additional items related to oscillation detection. First is the obvious condition that the sampling interval must be shorter than half the oscillation period. For the level of detail required in this paper integration periods of about one minute are required. This is the only obvious change to plans for FRESIP to support asteroseismology: for at least a subset of the brighter stars an observing cadence of one minute should be adopted. The second item to consider is how long the observing runs must be to return useful information. For the stars of M67 (F-G subgiants) the smallest mode frequency separations of $\sim 3~\mu \rm Hz$ would require a five day time base to safely resolve. In general much longer time bases would be preferred in order to support higher precision oscillation frequency determinations.

An additional consideration is what generates the network requirement. The principle feature characterizing multi-mode oscillations is the mode separations in frequency space. This is of both theoretical interest and observational convenience given that the repeated structures in a power spectrum can be exploited (via power spectra of power spectra techniques) to show the presence of weak signals, none of which could be believed individually. For the Sun this mode separation is 68 μ Hz; this can be resolved with observations spanning $\sim 1/68 \mu$ Hz = 4.5 hours. Stars of still roughly solar type, e.q. the upper main sequence and subgiants of M67 selected to have expected oscillation amplitudes of 3 to 5 times solar have mode separations of 20 to 40 μ Hz. At 40 μ Hz the observation window required for frequency resolution is 7 hours, at 20 µHz the required window is 14 hours. Observations obtained from a single site with nightly eight hour observing windows will marginally support detection of splittings at 40 μ Hz, but do not carry any information at 20 μ Hz, at least for the straight forward power spectrum of power spectrum analysis technique. Nightly eight-hour windows with 16 hour gaps carry this autocorrelation type of information for frequencies between 1/8 hours $\sim 35 \mu Hz$ out to the Nyquist frequency of the individual observations (8.3 mHz if a one minute cadence is assumed). Frequencies from 1/16 hours ~ 17 μHz (time from end of one window to start of the next) down to the inverse of the total time base may also be resolved. But the frequency desert domain of 17 to 35 μ Hz is not sampled by nightly observations with eight-hour windows no matter how many nights from a single site are strung together. This is not to claim that no information on modes separated by 17 to 35 μ Hz is contained in data acquired with nightly eight-hour windows. δ Scuti stars (see, e.g., Gilliland and Brown 1992b) often show individual modes with well determined frequency differences between 17 and 35 μ Hz, but this is for high signal-to-noise data analyzed via direct sinusoidal fits. For the low signal-to-noise data certain to hold

for solar-like oscillation experiments, the observations must be structured so as to remove the frequency desert associated with gapped data. In a formal sense the frequency gap for power spectra of power spectra disappears for 12 hour and greater windows. In practice the relative sensitivity for splitting in the general frequency domain of 20-40 μ Hz improves rapidly for windows of 12 to 18 hours with gains still to be made through elimination of all gaps.

Stars with mode separations favorable to detection with observations feasible from single sites (e.g. close solar analogues) have such small predicted amplitudes that even a 10-m telescope would not yield good enough precision. Stars with more favorable amplitude predictions (e.g. subgiants) have mode separations that require nearly continuous network campaigns. For the detection of weak oscillations characterized by frequency separations of 20-40 μ Hz a nearly complete longitude distributed network is a practical necessity. Lunar based observations could be ideal for the study of solar-like oscillations. Single nightly transits could extend to over 12 days allowing for frequency resolution of less than 1 μ Hz which is a good match to desired precisions for asteroseismology of solar-type stars.

3. Results From 4-m Network Campaign (Jan '92)

Given the need for several 4-m class telescopes observing in concert for about a one-week period, much effort was devoted to background studies and presentation of fundamentals. Gilliland and Brown (1992a) describe the work done to justify devoting some thirty nights of allocated 4-m time to this single observational project. Previous experiments on 1-m and 2-m class telescopes had shown that CCD ensemble photometry could deliver atmospheric scintillation plus Poisson statistics limited precision for relative time-series photometry of a stellar ensemble. A minor extrapolation from empirically obtained and theoretically understood results showed that 4-m observations should allow precisions of $\sim \! 300~\mu \rm mag$ for one minute integrations for stars of $m_B \leq 13.0$ as averaged over eight hour observing windows.

The 4-m network that was realized for January 1992 consisted of seven sites (two were only 2.5-m class telescopes and therefore would yield lower precision per unit time) for a total of 34 nights. With reasonable assumptions about weather losses (assume $\sim\!60\%$ clear) such a campaign was predicted to support detection levels corresponding to 16 μ mag (best cases), or about four times larger amplitude than the Sun.

The selection of a stellar ensemble to observe was driven by observational constraints; in particular the best possible match to the (Cassegrain mounted 512×512 T5HA CCD covering a field of 95 arcsec) setup at Kitt Peak was sought. The dipper asterism field of M67 contains eleven stars of 12th and 13th magnitude in B within an 80 arcsec square and has few fainter background contamination stars. Also important is that the eleven stars are mutually well separated (12" nearest neighbor) and have similar colors. Based on detailed searches this is believed to be the best field in the sky from purely observational constraints. From a theoretical perspective this is also an ideal field. All of the ensemble stars are high probability members of the cluster and M67 is one of the most thoroughly observed clusters – a necessary condition to support full

utility of any success at quantifying oscillation frequencies for some of the stars.

The observing campaign from 12-18 January 1992 resulted in a total of 156 hours of time series observations on the M67 ensemble over 22 separate telescope nights. The time-weighted mean aperture size contributing to the time series was 4.04 m. Sites returning useful data were: the 4-m at Kitt Peak, the 5-m at Palomar, the 3.6-m CFHT, the 3.9-m Anglo Australian Telescope, the 3.5-m at Calar Alto in Spain, and the 2.5-m Nordic Optical Telescope in the Canaries. The longitude distribution was sufficient to provide time-series coverage 64% of the time over a six-day period. During good observing conditions precisions of $\sim 300~\mu \rm mag$ each minute were reached, as expected.

Discussion of two special problems, one recognized before and the other after data was obtained, may be used to illustrate the special care required to reach the high precisions needed for this project.

Before the run it was recognized that differences of equipment and setup across the sites demanded thorough development of observing plans. At Kitt Peak the CCD had 0.18" (27 μ) pixels with deep (5 \times 10⁵ e^{-}) linear wells. The brightest star would reach an ideal level of about half saturation at peak intensity for nominal 1.5" seeing and a 60 second integration time. At CFHT the CCD had 0.2" (15 μ) pixels with a shallow well depth – not uncommon 0.6" CFHT seeing would push the saturation limit with exposure times of only ~1 second. With a 14 second readout overhead a disastrous duty cycle of $\sim 7\%$ would result, i.e., most of the time that should be spent averaging down atmospheric scintillation fluctuations and collecting photons to minimize the Poisson noise would actually be spent cycling the CCD. Under such conditions the data collected from CFHT (nominally expected to be the best site based on projections for atmospheric scintillation) would not contribute in a meaningful way. An obvious solution was to defocus the images at CFHT, allow much longer integrations and thus restore a good duty cycle. But too much defocus would blend the stellar images together and compromise the intensity extraction estimates. In order to use the correct defocus we generated a predictive error budget for each site as a function of any selectable (CCD options, f-ratio, etc.), or adjustable (defocus, integration time) options and solved for the set providing the smallest time series errors. Part and parcel of this process was empirical testing on a setup field before M67 could be observed at the beginning of nights.

After the fact a subtle nonlinearity at low intensity was recognized in the CCD used on the Palomar 5-m. The nonlinearity was such that for an input level of 1000 detected photons the signal level was ~10% lower (in a relative sense) than for an input signal 10 times larger. At lower intensity levels the effect was relatively larger, at levels above 5000 photons linearity was maintained. Due to fluctuations in low-intensity components of the images (from sky and seeing) this subtle low-intensity nonlinearity resulted in a much increased error budget (factor of two) for the Palomar time series. Once recognized it was possible to correct for this in the CCD reduction phase and reach final precision levels as expected.

A detailed analysis of the error budget showed that under good conditions the time series noise level varied in a way which followed the predictions for atmospheric scintillation plus Poisson noise limited data. As an example, the 4th brightest star in the ensemble showed a time series standard deviation (data from one night) of 252 ppm over the three hours at minimum airmass (77 second integrations), compared to a direct simulation of 238 ppm based on atmospheric scintillation, Poisson object noise and sky plus CCD readout contributions added in quadrature. At the three hours of highest airmass the observed standard deviation was 362 ppm (85 second integrations) versus a modeled value of 387 ppm (the latter suggests atmospheric scintillation probably increased more slowly than airmass to the 1.75 power assumed). In times of poor and variable seeing, or with variable transmission, the errors would exceed the limiting levels, but degradation with poor conditions was very modest. CCD ensemble photometry allows atmospheric scintillation plus Poisson statistics limits to be reached on 4-m telescopes and does so in a robust way that continues to provide excellent results even when conditions are far from photometric in quality.

Although the network campaign provided an immense amount of data (~8000 data points on 11 stars each with mean noise levels of about 300 to 500 ppm over the full time series), the search for oscillations is difficult. At best we might expect a S/N per data point of about 0.1; a time series plot will look like pure noise even if such a coherent signal is present. Through power spectra analyses the evidence for such signals can be brought out.

Solar observations show that many modes are simultaneously excited. The independent modes tend to be evenly spaced in the frequency domain (this is theoretically understood as individual modes differing by single steps of a high radial overtone quantum number) creating a picket fence effect (e.g. see Toutain and Fröhlich 1992) in power spectra. Searching for evenly spaced modes in the power spectra can provide another handle on detecting oscillations and this spacing is the theoretically interesting quantification (to lowest order) of the oscillations.

The full process of CCD data reduction, intensity extraction, massaging of time-series data, and power spectrum analysis is quite complicated and not amenable to providing direct 'error bars' on results. In all analyses we rely heavily on simulations to know: If a signal of a certain amplitude were present would we detect it? If no signal is present will we reach a null conclusion?

The following conclusions may be drawn from detailed analysis of the real data in comparison with hundreds of realistic simulations:

1. In no cases are stellar oscillations detected unambiguously.

A working definition of unambiguous: no reasonable scientist would doubt the basic detection. The signal would be obvious when analyzed in an appropriate way and this level of confidence is required to be of utility in challenging theory.

2. In the two stars with the lowest noise, multi-mode oscillations with peak amplitudes of 25 and 28 ppm respectively would have been unambiguously detected.

This result involves a substantial number of reasonable assumptions regarding the simulations used for 'calibration.' With this caveat in mind firm upper limits of 25 and 28 ppm (6-7 times solar) may be placed on oscillations in the two best cases. This result is just at the margin of being interesting to theorists, *i.e.*, it seems unlikely that amplitudes are

larger than suggested by simple theory (Christensen-Dalsgaard and Frandsen 1983), but the upper limits are not in conflict with expectations.

3. About half of the ensemble stars show evidence of oscillations with a distinct tendency for the inferred frequencies to be in good agreement with theory. The suggested amplitude of peak oscillations for these cases is about 20 ppm.

Unfortunately, detailed Monte Carlo style simulations show that similarly suggestive evidence would appear about 5% of the time given time series of pure noise analyzed in the same way. The significance of positive detections is therefore about 2σ .

4. Advanced Network Campaign Possibilities

The 4-m network campaign realized in January 1992 was sufficient to reach precision levels of interest for detecting oscillations on the stars in question. Oscillations may have been seen, but if so only at significance levels too low to support challenges to theory. Given the cost in terms of telescope time and the organizational effort of setting up such a campaign, a mere repeat is probably not justified.

It seems likely based on current theory that 100% improvement in the observational sensitivity would support unambiguous success for most of the M67 ensemble stars. What changes to the realized network would be required to yield a factor of two gain for the 'best' star in the M67 ensemble? What changes would be required to yield an across the board factor of two gain in sensitivity?

The 4-m network campaign included telescopes at Kitt Peak, Palomar, Mauna Kea, Australia, Spain, and the Canary Islands. The latter two sites contributed generally low importance data (the site in Spain contributed only part of one night, the site in the Canaries was 'only' a 2.5-m telescope). Although observations had a 64% filling factor over six days, or \sim 15 hour windows on average, the data from the Eastern flank was relatively weak compared to that from the American southwest. The actual network yielded good data on 20 full telescope nights. The star showing the best evidence for oscillations had a predicted (and at 2σ detected) frequency separation of adjacent modes of ~ 19 μ Hz. Simulations show that the oscillation characteristics expected for this star are particularly sensitive to the network data distribution. Adding five (summer) nights from the CTIO 4-m in Chile, three nights from the 4.2-m WHT in the Canaries, and two nights from the Russian 6-m improved sensitivity for mode separations at 19 μ Hz by a full 100%! From simple signal-to-noise considerations adding 10 nights to the already existing 20 nights would result in a $(3/2)^{1/2}$, or about 22% gain. But by adding the additional time at ideal longitudes the gain is a full 100%, a doubling of sensitivity for only a 50% augmentation. For ensemble stars with expected frequency separations of about 40 µHz the gain would be about 30% with most of the improvement following from simple $N^{1/2}$ considerations.

The ensemble stars with expected mode separations of $\sim 35\text{-}40~\mu\text{Hz}$ require primarily a brute force addition of data for improved sensitivity. Adding in seven nights each from the Keck 10-m and the MMT 6.5-m upgraded telescopes

would allow a factor of two sensitivity increase for these stars.

A network campaign consisting of the six sites yielding data in January 1992, plus the CTIO 4-m, WHT 4.2-m, Russian 6-m, Keck 10-m and MMT 6.5-m – all with seven night allocations – would better than double the sensitivity gains for all the M67 ensemble stars. With such a campaign unambiguous success capable of supporting a real challenge to stellar structure and evolution theory would be expected.

Successfully justifying a simultaneous one-week allocation on most of the world's largest telescopes would be no mean feat. The science to be derived from observations of a coeval M67 population might well justify such an observing campaign; any results would likely be complementary to what would follow from early space observations. However, in a wider scope the opportunities provided by space observations would far surpass what can be contemplated from the ground.

5. What Could be Done With FRESIP?

The prospects of pursuing photometric detection of stellar oscillations from space is excellent. Even a modest 1-m class telescope in space can easily outperform networks of much larger telescopes operating from the ground. The fundamental noise limitation from atmospheric scintillation does not exist for space observations. Placed in a suitable orbit a space observatory may obtain continuous monitoring for very long periods of time – conditions that are ideal for asteroseismology. Given the existing plans for FRESIP, modifications to allow stellar oscillation detection could be included with little additional cost or complexity.

The FRESIP project is imagined to be a 1.0 to 1.5-m very wide-field (~100 square degrees) telescope with several large format CCDs covering the focal plane. The images for some 8,000 program targets would be defocused to allow excellent signal-to-noise at very high count rates - ideal for asteroseismology. The CCDs would be read out at a rapid (few second) cadence with on-board processing providing summed intensities once per hour for previously identified targets to support planetary identification via periodic transit signals. This is a demanding, but potentially very powerful approach, that requires continuous viewing of the same field for at least three years - again ideal for asteroseismology. To provide excellent results for stellar oscillations would simply require that the signals for a subset of the stars be compiled on one minute intervals. Although elimination of noise from cosmic rays will be a challenge for on-board processing, it should be possible to maintain precisions that are close to the Poisson limit. Assuming CCDs with high efficiency, a 3000 Å wide filter and a 1-m telescope, then the count rate for FRESIP on an $m_B = 10.0$ star would be order 108 per minute for a Poisson noise of 100 ppm (ppm are equivalent to µmag used earlier to within 8%). At this precision (near the bright limit due to detector saturation) the solar amplitude could be marginally detected in about 12 days. Quantitatively robust frequency determinations would follow with data acquired on 10 times the temporal period (factor of three signal-tonoise gain) required for a simple marginal detection. Oscillation detection on early K dwarfs would require measurement of amplitudes a factor of five lower (Christensen-Dalsgaard and Frandsen 1983) than solar and thus require 25 times

as long for a basic detection. Following about 100 stars each of F, G, and K spectral types would allow invaluable statistics to be built up for the nature of p-modes on other stars (currently the Sun is the only solar-like star with a secure and quantitatively useful detection). The continuous monitoring would provide a perfect window function that is essential for robust interpretation of the data, but is so hard to come by with ground-based observations. The long time base would support both very precise frequency determinations and allow testing for frequency changes as may arise from stellar activity cycles. The asteroseismology data set that could result from FRESIP would provide fundamentally important information for the understanding of p-mode driving and for stellar structure and evolution theory.

6. Summary

I have argued that photometric detection of stellar oscillations on solar-like stars should be possible from the ground. However, a successful ground-based experiment is likely to require a heroic effort that cannot be repeated often and that will not extend over a sufficiently long time base to address many issues of interest. A network campaign conducted in January 1992 with 30 nights (average of 5 each at 6 sites) allocated was not sufficient to yield unambiguous results even on stars with favorably high predicted amplitudes. The 4-m campaign gave results a factor of three more sensitive than in any previous experiment. Further substantial gains will come only at a high cost in terms of telescope time. A network consisting of 10-12 of the worlds largest telescopes as should exist in 1997, all collaborating for 7 nights (i.e., 70-84 allocated nights at a mean aperture size over 5-m), would provide a further factor of two sensitivity gain. (The latter would be quite significant, since such would allow a prediction of clear success in detecting oscillations on several interesting stars.)

For completeness I should point out that Doppler measurements through high resolution spectroscopy is an additional technique applied to asteroseismology observations. Although the prospects are good for this technique to succeed in the near future, there are no unambiguous detections yet despite many efforts. This technique is most applicable to very bright stars. The need for network observations and long time bases will also limit results relying on spectroscopy.

Having spent several years pursuing very demanding ground-based observations directed toward stellar oscillation detection, I would like to comment on the prospects of conducting a FRESIP experiment from the ground. I would consider the potential for success at this to be even bleaker than for asteroseismology via photometry. The stellar oscillations have periods very short compared to nightly observing windows, we can therefore simply filter out or ignore any low frequency noise. The planetary detection experiment will be seeking signals with a characteristic time scale comparable to nightly observing windows as forced by the diurnal cycle. Under these circumstances the ground-based observations would be inherently impractical for the photometric detection of small planets. With both photometric and spectroscopic data of high quality, and well determined noise properties on time scales of less than one hour, I have attempted (with dismal results) to derive useful information on longer time scales. I believe a FRESIP experiment on the ground is not possible.

With only minor modifications to the proposed FRESIP mission excellent auxiliary science could be expected for asteroseismology. Indeed FRESIP is close enough to ideal for asteroseismology that similar experiments might be proposed with stellar oscillations as the primary science driver.

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